Chapter 18
Gordon decomposition of the vector/axial currents
–
from my book:
Understanding Relativistic Quantum Field Theory

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Chapter 18

Gordon decomposition of the vector/axial currents

18.1 Phase gradient versus Magnitude gradient

Wave-functions are constructed from complex amplitudes with both a phase ϕ and an absolute amplitude a. Both components play different roles which we want to study in more detail. We want to separate certain quantities into phase related and magnitude related components. Important in this respect will be the Gordon decomposition which separates the vector current in a charge and a spin related component.

In this section we use a more elementary approach using the Klein Gordon equation. Each component of the Dirac/Weyl spinor obeys separately also the Klein Gordon equation. To show this first we start of with the Dirac equation.

$$i\begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0 \end{pmatrix}\partial_\mu \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = m\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{18.1}$$

We now square the operator on the left hand side. In the first order equation the ψ_L and ψ_R chiral components are each the source of the other coupled via the constant m. Applying the operator twice couples ψ_L and ψ_R back into an expression where they each are their own source.

$$\left[\, i\begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0 \end{pmatrix}\partial_\mu \right]^2 \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = m^2\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{18.2}$$

The result is that ψ_L and ψ_R are given by independent equations. From the commutation rules of the Pauli matrices we derive.

$$\begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma^\nu \\ \tilde{\sigma}^\nu & 0 \end{pmatrix} \quad = \quad \begin{cases} I & \text{if } \mu = \nu \\ 0 & \text{if } \mu \neq \nu \end{cases} \tag{18.3}$$

Which shows that all four components obey the Klein Gordon equation.

$$\Box\, \psi\, I = -m^2 \psi\, I \tag{18.4}$$

This makes if physically meaningful to study the relating between the phase gradient ϕ_μ and the gradient of the absolute amplitude a in the context of the simpler Klein Gordon equation first.

We can describe wave-function around an arbitrary point which we choose as the origin of a coordinate system x^μ as an exponential with phase change rate ϕ_μ and absolute amplitude change rate a^μ. where ϕ_μ and a^μ are four-vector functions depending on x^μ.

$$\psi \; = \; e^{(a_\mu + i\phi_\mu)x^\mu} \tag{18.5}$$

The Klein Gordon equation becomes.

$$\left(\, \partial_\mu \partial^\mu + m^2 \, \right) e^{(a_\mu + i\phi_\mu)x^\mu} \; = \; 0 \tag{18.6}$$

We evaluate this and then set $x^\mu = 0$ because we are interested in this particular point only. This gives us the following expression.

$$\left(a_\mu a^\mu \; - \; \phi_\mu \phi^\mu \; + \; 2i \; a_\mu \phi^\mu \; - \; 2i \; \partial^\mu \left(a_\mu + i\phi_\mu \right) \; \right) \psi \; = \; 0 \tag{18.7}$$

We can spit this expression into two independent equations, one for the real and one for the imaginary part.

$$\begin{array}{lcll} \phi_\mu \phi^\mu - m^2 & = & a_\mu a^\mu + 2 \; \partial_\mu a^\mu & \text{real part} \\ \partial_\mu \phi^\mu & = & -a_\mu \phi^\mu & \text{imaginary part} \end{array} \tag{18.8}$$

If $a^\mu = 0$ then this simplifies to the following two equations.

$$\begin{array}{lcll} \phi_\mu \phi^\mu - m^2 & = & 0 & \text{Momentum space Klein Gordon eq.} \\ \partial_\mu \phi^\mu & = & 0 & \text{Continuity relation} \end{array} \tag{18.9}$$

The first one is simply the Klein Gordon equation in momentum space. It is however also locally true, that is, energy and momentum are allowed to be locally different as long as the equation holds in each point. The second equation is the continuity equation for energy/momentum or equivalently, the charge/current density which transforms in the same way.

18.2 Gordon decomposition of the vector current

We now precede with the Gordon decomposition of the vector current of the Dirac equation. It splits the vector current J_V^μ into a current related to the charge and a current related to the spin. The vector current, normalized as a charge/density current is.

$$j_V^\mu \;=\; -\frac{e}{m}\;\bar{\psi}\gamma^\mu\psi \tag{18.10}$$

Where the sign of e determines the sign of the electric charge. We have divided by mc and then multiplied by ec to go from the momentum four-vector to the charge/current density.

We will split this current into one (J_ϕ^μ) which depends on the phase change rates but which is independent of the changes of the magnitude, and another (J_a^μ) which does not depend on the phase change rates but only on the changes (derivatives) of the magnitude.

$$j_V^\mu \;=\; J_\phi^\mu \;+\; J_a^\mu \tag{18.11}$$

This is done by either adding or subtracting a complex conjugate term.

$$\begin{aligned} \phi &= \tfrac{1}{2i}\Big(\;(a+i\phi)\;-\;(a+i\phi)^*\;\Big) \\ a &= \tfrac{1}{2}\;\Big(\;(a+i\phi)\;+\;(a+i\phi)^*\;\Big) \end{aligned} \tag{18.12}$$

We achieve this by first splitting the vector current into two equal parts.

$$j_V^\mu \;=\; -\frac{e}{2m}\Big(\;\bar{\psi}\gamma^\mu\psi \;+\; \bar{\psi}\gamma^\mu\psi\;\Big) \tag{18.13}$$

We replace ψ in the first term above with the help of the Dirac equation, and we replace $\bar{\phi}$ with the use of the complex conjugate of the Dirac equation. The Dirac equation and its complex conjugate are.

$$i\gamma^\nu\partial_\nu\psi \;= mc\,\psi, \qquad\quad -\,i\gamma^\nu\partial_\nu\bar{\psi} \;= mc\,\bar{\psi} \tag{18.14}$$

The γ^ν matrix is just a constant for differentiation so we can move it through the ∂_ν operator, we get.

$$j_V^\mu \;=\; -\frac{ie\hbar}{2m^2c}\left(\;\bar{\psi}\gamma^\mu\gamma^\nu\frac{\partial\psi}{\partial x^\nu}\;-\;\frac{\partial\bar{\psi}}{\partial x^\nu}\gamma^\nu\gamma^\mu\psi\;\right) \tag{18.15}$$

These two terms now *subtract* if $\mu=\nu$ and they *add* if $\mu\neq\nu$ because.

$$\begin{array}{rcrl} \gamma^\mu\,\gamma^\nu & = & \gamma^\nu\,\gamma^\mu & \qquad\text{if } \mu=\nu \\ \gamma^\mu\,\gamma^\nu & = & -\gamma^\nu\,\gamma^\mu & \qquad\text{if } \mu\neq\nu \end{array} \tag{18.16}$$

The current related to the phase change rate becomes (using $\gamma^0\gamma^0=1$ and $\gamma^i\gamma^i=-1$)

$$j_\phi^\mu \;=\; -\frac{ie\hbar}{2m^2c}\left(\frac{\partial\bar{\psi}}{\partial x_\mu}\psi-\bar{\psi}\frac{\partial\psi}{\partial x_\mu}\right) \tag{18.17}$$

We recognize this as the expression for the charge/current density of the Klein Gordon equation. Subsequently, the current which depends on the magnitude changes is.

$$j_a^\mu \;=\; -\frac{e\hbar}{m^2c}\;\frac{\partial}{\partial x^\nu}\left(\;\bar{\psi}\,\sigma^{\mu\nu}\,\psi\;\right) \tag{18.18}$$

Where we have used the definition $\sigma^{\mu\nu}=\frac{i}{2}\left(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu\right)$. The derivative here is acting on the whole bilinear $\bar{\psi}\sigma^{\mu\nu}\psi$, from which the phase information is already eliminated.

We showed in section **??** that the bilinear $\bar{\psi}\sigma^{\mu\nu}\psi$ transforms as the electromagnetic field tensor, or equivalently, the polarization/magnetization tensor. Indeed, when we recall Maxwell's inhomogeneous equations in tensor form. (with ϵ_o, μ_o and c set to 1)

$$j_\phi^\mu \;=\; -\partial_\nu\;F^{\mu\nu} \;=\; -\frac{\partial}{\partial x^\nu}\left[\begin{array}{cccc} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{array}\right] \tag{18.19}$$

We see that the divergence of the rows does lead to a charge/current density. This means that we can define $\bar{\psi}\sigma^{\nu\mu}\psi$ as an electromagnetic field tensor tensor. In the rest frame $\bar{\psi}\sigma^{\nu\mu}\psi$ becomes.

$$\bar{\psi}\sigma^{\mu\nu}\psi\,\bigg|_{\beta=0} = \frac{\bar{\psi}\psi}{2}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & i\hat{s}_z & -i\hat{s}_y \\ 0 & -i\hat{s}_z & 0 & i\hat{s}_x \\ 0 & i\hat{s}_y & -i\hat{s}_x & 0 \end{bmatrix} \tag{18.20}$$

Where $\hat{s}$ is the unit spin vector. Thus, there is only a magnetic field in the rest-frame. Under an arbitrary boost $\vec{\beta}$ the tensor becomes.

$$\bar{\psi}\sigma^{\mu\nu}\psi \ =$$

$$\frac{\bar{\psi}\psi}{2}\begin{bmatrix} 0 & -(\ \hat{s}_{\otimes}\,\beta\gamma\ )_x & -(\ \hat{s}_{\otimes}\,\beta\gamma\ )_y & -(\ \hat{s}_{\otimes}\,\beta\gamma\ )_z \\ (\ \hat{s}_{\otimes}\,\beta\gamma\ )_x & 0 & i(\hat{s}_{\|}+\hat{s}_{\perp}\gamma)_z & -i(\hat{s}_{\|}+\hat{s}_{\perp}\gamma)_y \\ (\ \hat{s}_{\otimes}\,\beta\gamma\ )_y & -i(\hat{s}_{\|}+\hat{s}_{\perp}\gamma)_z & 0 & i(\hat{s}_{\|}+\hat{s}_{\perp}\gamma)_x \\ (\ \hat{s}_{\otimes}\,\beta\gamma\ )_z & i(\hat{s}_{\|}+\hat{s}_{\perp}\gamma)_y & -i(\hat{s}_{\|}+\hat{s}_{\perp}\gamma)_x & 0 \end{bmatrix} \tag{18.21}$$

The first column and row now contain the E components which correspond to the transformed B field components. We note that the B field components are imaginary while the E field components are real. This leads us to define the electromagnetic field using complex numbers:

$$\vec{F} \ = \ \mathsf{E} \ + \ i\mathsf{B}, \qquad \text{or} \qquad \vec{F} \ = \ \mathsf{E} \ + \ \tilde{\mathsf{B}} \tag{18.22}$$

That is, we have to multiple the B fields with an extra factor i in order to obtain the correct electromagnetic tensor.

$$F^{\mu\nu} \;=\; -\frac{e\hbar}{m}\,\bar{\psi}\sigma^{\mu\nu}\psi \;=\; \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -\tilde{B}_z & \tilde{B}_y \\ E_y & \tilde{B}_z & 0 & -\tilde{B}_x \\ E_z & -\tilde{B}_y & \tilde{B}_x & 0 \end{bmatrix} \tag{18.23}$$

This expression corresponds with the usual inhomogeneous Maxwell equations given in SI by.

$$\rho \;=\; \epsilon_o\,\nabla\cdot\mathsf{E}, \qquad \vec{j}_a \;=\; \frac{1}{\mu_o}\,\nabla\times\tilde{\mathsf{B}} \;-\; \epsilon_o\,\frac{\partial\mathsf{E}}{\partial t} \tag{18.24}$$

$$\rho \;=\; \nabla\cdot\mathsf{D}, \qquad \vec{j}_a \;=\; \nabla\times\tilde{\mathsf{H}} \;-\; \frac{\partial\mathsf{D}}{\partial t} \tag{18.25}$$

$$\rho \;=\; \nabla\cdot\mathsf{P}, \qquad \vec{j}_a \;=\; \nabla\times\tilde{\mathsf{M}} \;-\; \frac{\partial\mathsf{P}}{\partial t} \tag{18.26}$$

Using either the E and B fields or the D and H fields. We will however use the polarization vector P and the magnetization vector $\tilde{\mathsf{M}}$ as the physical interpretation, where the spin is considered to give rise to the magnetization of the wave function. $\tilde{\mathsf{M}}$ is strictly zero outside the wave-function unlike $\tilde{\mathsf{H}}$ and $\tilde{\mathsf{B}}$.

18.3 Effective spin-current around the wave-packet

Figure 18.1 shows the axial current of a Gaussian like shaped wave packet at the left and the corresponding effective current at the right. Recalling Stokes law we see the correspondence between the two.

In the center, the axial circular currents cancel each other out. At the edge, where there is a gradient, an effective current arises as a result. This effect also occurs in magnetic media where an effective current occurs due to the change in magnetization. $\vec{j} = \nabla\times\mathsf{M}$

The quantity $\bar{\psi}\sigma^{\nu\mu}\psi$ is therefor best interpreted as the (relativistically transformed) magnetization of the electron's wave-function. Note that $\bar{\psi}\sigma^{\nu\mu}\psi$ is zero if there is no local charge density since it is proportional to the absolute square of the amplitude of the wave function.

These insights are due to Belinfante [?] (1939), JJ.Sakurai [?] (1967) and H.Ohanian [?] (1986).

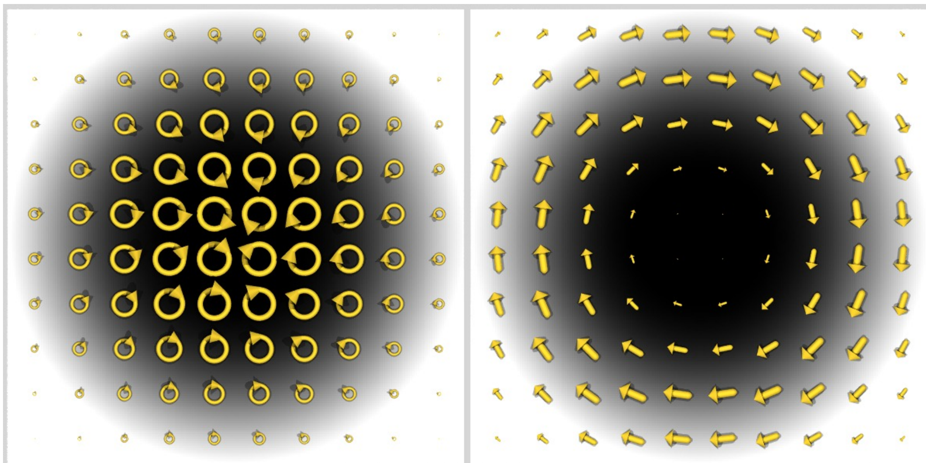

Figure 18.1: Axial and effective current density of a Gaussian packet

From equation (18.20) we see that, in the rest-frame, the axial current $J^{\mu}_{A} = \bar{\psi}\gamma_{\mu}\psi$ is proportional to the magnetization components of the tensor field $\bar{\psi}\sigma^{\nu\mu}\psi$. The two however transform differently under a boost.

$$\text{Total magnetic moment:} \quad \vec{\mu} \;=\; \int \mathsf{M}\; dS \;=\; \tfrac{1}{2}\int \vec{r}\times\vec{j}\; dS \qquad (18.27)$$

We can interpret the left side of figure 18.1 as the magnetization field M and the right side as the effective current $\vec{j}$=$\nabla\times\mathsf{M}$. (In the rest-frame). The total magnetic moment $\vec{\mu}$ can be derived from both as shown.

18.4 Charge/magnetic-moment ratio of leptons

The table shows the mass/spin ratios and the charge/magnetic-moment ratios for the three leptons. The electron, the muon and the tau-lepton. The ratios for the electron are normalized as 1:1.

These ratios are explained by the Gordon decomposition of the Dirac vector field, except for the small magnetic moment anomaly terms caused by interaction with virtual particle pairs.

The ratios correspond to the difference between the two currents j^μ_ϕ and j^μ_a. The first current does depend on the phase change rates while the second current doesn't.

leptons:	ELECTRON	MUON	TAU
Spin z-component Mass in MeV Mass ratio	1/2 $\hbar$ 0.510998918 1.000000000	1/2 $\hbar$ 105.6583692 206.7682838	1/2 $\hbar$ 1776.99 3477.48
Charge Magnetic moment Mag.moment ratio Magnetic anomaly	1.0 0.928476412e-23 1.00000000000000 1.00115965218085	1.0 4.49044799e-26 1/206.7669894 1.00116592080	1.0 2.6700053e-27 1/3477.43 1.00117324[1]

j^μ_ϕ corresponds with the energy/momentum vector as well as the charge/ current *density* vector, both transform in the same way.

Being dependent on the phase change rate of the wave functions means that they are larger by a factor proportional to the mass in the rest frame compared to the quantities which correspond to j^μ_a: The spin and the magnetic moment density.

[1]theoretical value from QED and QFT

18.5 Gordon decomposition of the axial current

Similar to the Gordon decomposition of the vector current, we will now handle the Gordon decomposition of the axial. current. The axial current is given by.

$$j_A \;=\; \frac{ie}{m}\;\bar{\psi}\gamma^\mu\gamma^5\psi \tag{18.28}$$

Where the sign of e determines the sign of the charge. We will again split this current into one ($j_{A\phi}$) which depends on the phase change rates but which is independent of the changes of the magnitude, and another (j_{Aa}) which does not depend on the phase change rates but only on the changes (derivatives) of the magnitude.

$$j_A \;=\; j_{A\phi} \;+\; j_{Aa} \tag{18.29}$$

The first step is to split the axial current into two equal parts,

$$j_A \;=\; \frac{ie}{2m}\bigg(\;\bar{\psi}\gamma^\mu\gamma^5\psi \;+\; \bar{\psi}\gamma^\mu\gamma^5\psi\;\bigg) \tag{18.30}$$

and then to replace replace ψ in the first term above with the help of the Dirac equation and to replace $\bar{\phi}$ with the use of the complex conjugate of the Dirac equation. The gamma matrices are just a constants for differentiation so we can move them through the ∂_ν operator, we get.

$$j_A \;=\; -\frac{e\hbar}{2m^2c}\bigg(\;\bar{\psi}\gamma^\mu\gamma^5\gamma^\nu\frac{\partial\psi}{\partial x^\nu} \;-\; \frac{\partial\bar{\psi}}{\partial x^\nu}\gamma^\nu\gamma^\mu\gamma^5\psi\;\bigg) \tag{18.31}$$

These two terms now *add* if $\mu=\nu$ and they *subtract* if $\mu\neq\nu$ because.

$$\begin{aligned} \gamma^\mu\gamma^5\gamma^\nu &= -\gamma^\nu\gamma^\mu\gamma^5 && \text{if } \mu=\nu \\ \gamma^\mu\gamma^5\gamma^\nu &= \;\;\;\gamma^\nu\gamma^\mu\gamma^5 && \text{if } \mu\neq\nu \end{aligned} \tag{18.32}$$

The signs are exactly the opposite of what they are in the case of the Gordon decomposition of the vector current, due to the way γ^5 operates. Now, the current which does only depend on the phase change rates and not on the magnitude change rates becomes.

$$j_{A\phi} \;=\; \frac{e}{m^2c}\left(\,\bar{\psi}\,\sigma^{\mu5\nu}\,\psi\,\right)\mathrm{p}_\nu \tag{18.33}$$

Where $\sigma^{\mu5\nu}$ is the dual tensor of $\sigma^{\mu\nu}$ and p_ν represents the phase change rates. Note that p_ν is covariant now $p_\nu = (\,p^t, -p^x, -p^y, -p^z\,)$ while it is contravariant in case of the vector current where the spatial components obtained an extra minus sign because of $\gamma^i\gamma^i = -1$ while $\gamma^0\gamma^0 = 1$

The part of the axial current which depends only on the changes of the magnitude and not on the phase change rates is.

$$j_{Aa} \;=\; \frac{e\hbar}{2m^2c}\;\frac{\partial}{\partial x_\nu}\left(\,\bar{\psi}\gamma^5\psi\,\right) \tag{18.34}$$

Where $\bar{\psi}\gamma^5\psi$ is zero under quite general conditions for a free electron. This leaves as $j_{A\phi}$ as the basic contributor to the axial current j_A. We have defined the dual tensor $\sigma^{\mu5\nu}$ as.

$$\sigma^{\mu5\nu} \;=\; \frac{i}{2}\left[\,\gamma^\mu\gamma^5, \gamma^\nu\,\right], \quad \text{with} \;\; \left[\gamma^\mu\gamma^5, \gamma^\nu\right] = \left(\gamma^\mu\gamma^5\gamma^\nu - \gamma^\nu\gamma^\mu\gamma^5\right) \tag{18.35}$$

or, written out in full for your convenience:

$$\sigma^{\mu5\nu} \;=$$

$$\begin{bmatrix}
\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} -i\sigma^x & 0 \\ 0 & -i\sigma^x \end{bmatrix} & \begin{bmatrix} -i\sigma^y & 0 \\ 0 & -i\sigma^y \end{bmatrix} & \begin{bmatrix} -i\sigma^z & 0 \\ 0 & -i\sigma^z \end{bmatrix} \\
\begin{bmatrix} i\sigma^x & 0 \\ 0 & i\sigma^x \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} \sigma^z & 0 \\ 0 & -\sigma^z \end{bmatrix} & \begin{bmatrix} -\sigma^y & 0 \\ 0 & \sigma^y \end{bmatrix} \\
\begin{bmatrix} i\sigma^y & 0 \\ 0 & i\sigma^y \end{bmatrix} & \begin{bmatrix} -\sigma^z & 0 \\ 0 & \sigma^z \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} \sigma^x & 0 \\ 0 & -\sigma^x \end{bmatrix} \\
\begin{bmatrix} i\sigma^z & 0 \\ 0 & i\sigma^z \end{bmatrix} & \begin{bmatrix} \sigma^y & 0 \\ 0 & -\sigma^y \end{bmatrix} & \begin{bmatrix} -\sigma^x & 0 \\ 0 & \sigma^x \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
\end{bmatrix} \tag{18.36}$$

The bilinear field $\bar{\psi}\,\sigma^{\mu\nu}\,\psi$ relates to the bilinear field $\bar{\psi}\,\sigma^{\mu 5\nu}\,\psi$ in the same way as the electromagnetic field tensor F relates to its dual, that is, the relationship,

$$\bar{\psi}\,\sigma^{\mu\nu}\,\psi \quad \Leftrightarrow \quad \bar{\psi}\,\sigma^{\mu 5\nu}\,\psi \tag{18.37}$$

is equivalent to the relationship between $F^{\mu\,\nu}$ and ${}^*F^{\mu\,\nu}$:

$$\begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z & -E_y \\ B_y & -E_z & 0 & E_x \\ B_z & E_y & -E_x & 0 \end{bmatrix} \tag{18.38}$$

$$j_{A\phi} \;=\; \frac{e}{\epsilon_o}\begin{bmatrix} 0 & -c\tilde{M}_x & -c\tilde{M}_y & -c\tilde{M}_z \\ c\tilde{M}_x & 0 & P_z & -\,P_y \\ c\tilde{M}_y & -\,P_z & 0 & P_x \\ c\tilde{M}_z & P_y & -\,P_x & 0 \end{bmatrix}\begin{bmatrix} j^o \\ -j^x \\ -j^y \\ -j^z \end{bmatrix} \tag{18.39}$$

$$\mathcal{S}_\mu \;=\; \frac{e}{\epsilon_o}\begin{bmatrix} 0 & -\,\mathsf{H}_x & -\,\mathsf{H}_y & -\,\mathsf{H}_z \\ \mathsf{H}_x & 0 & \mathsf{D}_z & -\,\mathsf{D}_y \\ \mathsf{H}_y & -\,\mathsf{D}_z & 0 & \mathsf{D}_x \\ \mathsf{H}_z & \mathsf{D}_y & -\,\mathsf{D}_x & 0 \end{bmatrix}\begin{bmatrix} A^o \\ -A^x \\ -A^y \\ -A^z \end{bmatrix} \tag{18.40}$$

$$\mathcal{F}^\mu \;=\; \frac{e^2}{4\pi^2}\,\varepsilon^{\mu\alpha\beta\gamma}\;F^{\mu\alpha}A_\alpha \tag{18.41}$$

$$j_{A\phi} \;=\; -\mathcal{F}^{\mu\nu}\;j_\nu\;+ \tag{18.42}$$

We recognize this as the expression for the charge/current density of the Klein Gordon equation. Subsequently, the current which depends on the magnitude changes is.

$$j_{Aa} \; = \; \frac{e\hbar}{2m^2c} \;\; \frac{\partial}{\partial x_\nu}\left(\; \bar{\psi}\gamma^5\psi \; \right) \tag{18.43}$$